Relation Between Transmission and Throughput of Slotted ALOHA Local Packet Radio Networks

Ker Zhang and Kaveh Pahlavan, *Senior Member, IEEE*

***Abstract*— A new method for the exact calculation of the throughput of a centralized slotted ALOHA packet radio network over slow Rayleigh-fading channels is presented and the results are compared with the computer simulations. Also, upper and lower bounds on the performance are provided. The effects of capture on the throughput of the system are related to the modulation and coding technique, signal to noise ratio of the received signal, general terminal distribution in the area and the length of the transmitted packets. The BPSK, coherent and noncoherent binary FSK modulations and BCH coding are considered for the exact calculations. The results of the analysis shows that the maximum average throughput of the slotted aloha packet radio network is around 60% rather than 36% predicted from the simplified analysis. In contrast with some previous reports, it is shown that the throughput of the system is not affected significantly by the use of coding or the change of packet lengths.**

I. Introduction

RECENTLY, local wireless communication networks have attracted considerable attention [1], [2] and various aspects of indoor radio propagation [3]–[5], transmission [6], [7] and networking [8] are under investigation. This paper presents a new method for the calculation of the average throughput of the centralized slotted ALOHA local packet radio networks which relates the throughput to the specifics of the transmission system and the fading characteristic of the radio channel. It is well known that the throughput of an ideal slotted ALOHA system where none of the packets survive collisions, is bounded to 0.36 [9]. In a centralized packet radio enviornment, however, some packets do survive the collision, resulting in an increase in the throughput. In the literature, this phenomenon is referred to as capture. In the fading multipath radio channels, the effects of capture are related to the details of the media, which makes capture a function of the modulation and coding technique used for transmission, characteristics of the fading, length of the packets and the distribution of the location of the terminals transmitting to the central station.

Recently, there have been efforts to analyze the effects of capture in packet radio systems using various assumptions. The analysis in [10]–[12], forms the ratio of the received power from two terminals and compares it with a parameter referred to as the capture parameter. If the ratio is higher than the capture parameter, the packet with higher received power survives the collision. The difference in the received power is due to the varying distances of the terminals from the central station, where the terminals are uniformly distributed in a local indoor area. The analysis does not include the effects of modulation technique, fading characteristic of the channel, and the length of the packets; the terminals are assumed to be uniformly distributed in the area surrounding the central station. The analysis in [13], [14] forms the ratio of the power of a test packet to the total sum of all other colliding packets and compares the result with the capture parameter. If the ratio is higher than the capture parameter, the test packet survives the collision. The channel is assumed to be Rayleigh [13] or log-normal [14] fading, and the analysis is presented for a general distribution function of the terminals. The effects of packet length and modulation techniques are not included in the work.

The above approaches isolate the collision mechanism from the modulation, coding technique, and the effect of the additive white Gaussian noise by defining a capture parameter based on the ratio of the received powers from various terminals. In reality, the capture of a packet is a random process which is a function of the modulation technique used for transmission, received signal-to-noise ratio, and the length of the packets. The received power is a function of terminal distribution and the channel characteristics between the central station and the terminals. An analysis for a system with uniformly distributed terminals in indoor radio channels is given in [15], which considers the modulation and coding in Rayleigh-fading environment. This analysis is based on the modeling of the capture presented in [17] and assumes the signal from the interfering packets to be Gaussian. As will be shown later, this assumption ignores the correlations between the interference noise for different bits in the same packet and provides a lower bound to the performance. A different approach to the problem is presented in [16], which analyzes the system for a general distribution function for the terminals in fast Rayleigh-fading channels.

This paper presents another approach to relate the capture effects in slotted ALOHA to the transmission system and the fading characteristics of the channel. The throughput is related to modulation and coding techniques, the length of the packets, signal-to-noise ratio, and distribution of the terminals. The exact solution and two bounds using Gaussian assumption are considered. These bounds define the area in which the exact solution lies.

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K. Zhang is with Rockwell International, Newport Beach, CA 92658.

K. Pahlavan is with the Department of Electrical Engineering, Worcester Polytechnic Institute, Worcester, MA 01609.

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Section II provides the absolute bounds for the throughput of the slotted ALOHA in presence of capture. The capture model for a slotted ALOHA system used in this paper is presented in Section III. Section IV obtains the probability of capture as a function of the number of interfering packets and the signal-to-noise ratio for the exact calculation and two bounds. Section V discusses the effects of various modulation and coding techniques. Results and discussions are provided in Section VI.

II. Absolute Bounds for Performance of Slotted ALOHA System

The system is assumed to be an ideal slotted ALOHA network with a base station located in the center and terminals distributed around it with a given distribution. This system has negligible propagation delay, perfect acknowledgments from the receiver, and an infinite number of terminals.

For a slot size T and an average packet generation rate from all the terminals λ, the average number of packets arriving in a slot is $G = \lambda T$. When the arrival process is Poisson, the probability that N packets arrive in a slot is

$$R_N(G) = \frac{G^N}{N!} \exp(-G). \tag{1}$$

At the beginning of every slot, we assume there are $N+1$ packets generated from the terminals, one of these packets is randomly chosen to be the test packet which is locked to the receiver, while the other N packets are considered to be interference to the test packet. Defining $P_C(N)$ as the probability that the test packet is captured with N interfering packets, the averaged throughput of the system associated with this probability of capture is the average number of packets received successfully per time slot which is given by

$$S(G) = \sum_{N=0}^{\infty} R_{N+1}(G) P_C(N). \tag{2}$$

In the conventional calculations for the slotted ALOHA [18], it is assumed that at each collision all packets are destroyed and a packet survives at the receiver only if there is no collision. This implies $P_C(0) = 1$ and $P_C(N) = 0$, for $N \geq 1$, which lead to

$$S(G) = G \exp(-G), \tag{3}$$

for a conventional ALOHA. Equation (3) provides an absolute lower bound for the performance, if the transmission errors for a single packet are neglected, and is usually referred to as a case with no-capture. In reality, due to the transmission error, $P_C(0)$ is not exactly one. However, with packet error rates in the range from 10^{-3} to 10^{-5}, $P_C(0)$ can be assumed to be approximately one.

In the presence of capture, some of the packets involved in a collision will survive. Under an ideal situation, one packet survives a collision involved with $N+1$ packets. This case is referred to as the perfect capture for which $P_C(N) = 1$, for all values of N. Substituting $P_C(N) = 1$ in (2), perfect capture provides an upper bound for the throughput of the slotted ALOHA system with capture which is given by

$$S(G) = 1 - \exp(-G). \tag{4}$$

For large values of G, the throughput approaches 1 and the channel is fully utilized. The results provided in the rest of this paper are for practical consideration and should be compared with the two bounds provided in (3) and (4).

III. Mathematical Modeling of the Capture

In this section, we analyze the statistics of the parameters associated with the received signal in Rayleigh-fading channels with N interfering packets.

Under slow Rayleigh fading where the rate of variation of the channel is much slower than the data rate, all the bits in a received packet have the same energy. For Rayleigh fading, the received power from the terminals is exponentially distributed (the absolute value of their amplitudes is Rayleigh distributed) and the phase difference between the received signal and the local oscillator in the receiver is uniformly distributed. When the central station receives $N+1$ packets, one of these packets (test packet) is phase locked to the receiver and it has a zero phase. The bits in the other N packets are received with a uniformly distributed phase offset. The receiver decision variable for bit k in the test packet is [13]

$$\begin{aligned} r_{0k} &= \alpha_{0k} a_{0k} + \sum_{i=1}^{N} \alpha_{ik} \cos\theta_{ik} a_{ik} + \eta_{0k}; \\ &= \alpha_{0k} a_{0k} + \sum_{i=1}^{N} \alpha_{ik} g_{ik} + \eta_{0k} \\ &= \alpha_{0k} a_{0k} + a_{Nk} + \eta_{0k}; \qquad 1 \leq k \leq L \end{aligned} \tag{5}$$

where α_{ik} (the transmitted kth bit for packet i) is ± 1, a_{ik} (the amplitude of the received kth bit for the packet) is Rayleigh with average power of $\overline{p}_i, \theta_{ik}$ (the phase offset of the ith interfering packet with the test packet) is uniform and η_{0k} is the additive Gaussian noise with variance $N_0/2$. The average received power from a terminal $\overline{p}_i$ is a random variable with a probability density function determined by the terminal distribution in the area. The second term in the last line of (5) is the interference signal $a_{Nk} = \Sigma_{i=1}^{N} \alpha_{ik} g_{ik}$ where $g_{ik} = a_{ik} \cos\theta_{ik}$ is Gaussian distributed with variance $\overline{p}_i/2$. If the g_{ik}'s and α_{ik}'s for different bits are independent, a_{Nk} is a Gaussian random variable which takes an independent value for each bit in a packet. We refer to this case as the Gaussian assumption [15].

In a slow-fading channel, the effects of fading on all bits of a received packet is the same, and we assume $a_0 = a_{0k}$ and $g_i = g_{ik}$, for $1 \leq k \leq L$. Since α_{ik}'s are random binary ± 1 data which change from bit to bit, the interference signal $a_{Nk} = \Sigma_{i=1}^{N} \alpha_{ik} g_i$ randomly takes one of the 2^N possible combinations of g_i's. If the data pattern in the interference term is the same for all bits $a_{Nk} = a_N$ is a Gaussian random variable which takes the same value for all bits of a packet. For the Gaussian assumption, the variance of the a_{Nk} is $\frac{\overline{P}_N}{2}$ where $\overline{P}_N = \Sigma_{i=1}^{N} \overline{p}i$.

The distributions of a_0 and g_i are conditional Rayleigh and Gaussian, respectively, and for a given averaged power $\overline{p}i$ they are given by

$$f_{a_0}(a_0 \mid \overline{p}_0) = \frac{2a_0}{\overline{p}_0} \exp(-a_0^2/\overline{p}_0); \tag{6}$$

$$f_{g_i}(g_i \mid \overline{p}_i) = \frac{1}{(\pi \overline{p}_i)^{1/2}} \exp(-g_i^2/\overline{p}_i). \tag{7}$$

With Gaussian interference assumption, the distribution of a_{Nk} given $\overline{P}_N$ is also Gaussian

$$f_{a_{Nk}}(a_{Nk} \mid \overline{P}_N) = \frac{1}{(\pi \overline{P}_N)^{1/2}} \exp(-a_{Nk}^2/\overline{P}_N). \tag{8}$$

For the system considered in this paper, all the terminals have the same distribution of location and the distribution of the averaged received power of all the packets, $\overline{p}_i$ is the same. For a given distribution of terminals, we can obtain the distribution of $\overline{p}_i$ represented by $f_{\overline{p}_i}(\overline{p}_i)$, and the distribution of a_0 and g_i can be obtained from

$$f_{a_0}(a_0) = \int_0^\infty f_{a_0}(a_o \mid \overline{p}_0) f_{\overline{p}_0}(\overline{p}_0)\, d\overline{p}_0. \tag{9}$$

$$f_{g_i}(g_i) = \int_0^\infty f_{g_i}(g_i \mid \overline{p}_i) f_{\overline{p}_i}(\overline{p}_i)\, d\overline{p}_i. \tag{10}$$

With the Gaussian assumption, the distribution of $\overline{P}_N$ is the convolution of N distributions;

$$f_{\overline{P}_N}(\overline{P}_N) = f_{\overline{p}_1}(\overline{p}_1) * f_{\overline{p}_2}(\overline{p}_2) * \cdots * f_{\overline{p}_N}(\overline{p}_N)* \tag{11}$$

where "*" represents the convolution operator.

A. Calculation of Actual Distributions

The pdf of $\overline{p}_i$ is a function of the distribution of terminals and the distance-power relation for the received signal. We consider two distributions for the terminals. The first distribution corresponds to a case in which the average received power from a terminal is constant. The distribution of the average received power in this case can be given by

$$f_{\overline{p}_i}(\overline{p}_i) = \delta(\overline{p}_i - \overline{p}) \tag{12}$$

where $\overline{p}$ is the average received power for a terminal at the normalized unit distance from the base station. In practice, this case represents either a ring distribution in which all the terminals have the same distance from the base station, or the case with power control, when a central controller keeps the average received power from all terminals at the same level.

By substituting (6), (7), and (12) into (9) and (10), we have the following distributions for the amplitude of the test bit and the interference bits for the ring distribution:

$$f_{a_0}(a_0) = \frac{2a_0}{\overline{p}_0} \exp(-a_0^2/\overline{p}_0); \tag{13}$$

$$f_{g_i}(g_i) = \frac{1}{(\pi \overline{p}_0)^{1/2}} \exp(-g_i^2/\overline{p}_0). \tag{14}$$

For the Gaussian assumption, the distribution of $\overline{P}_N$ is determined by substituting (12) in (11)

$$f_{\overline{P}_N}(\overline{P}_N) = \delta(\overline{P}_N - N\overline{p}). \tag{15}$$

The second terminal distribution considered in this paper is a Bell shape distribution of the form $p(r) = 2r\exp(-\frac{\pi}{4}r^4)$ [r is the normalized distance between the transmitter and the receiver]. Assuming fourth power for the distance-power law, the average received power for a given distance r is $\overline{p}_i = r^{-4}$. In this case, distribution of the average received power is [13]

$$f_{\overline{p}_i}(\overline{p}_i) = \frac{1}{2} \overline{p}_i^{-3/2} \exp\left(-\frac{\pi}{4\overline{p}_i}\right). \tag{16}$$

Substituting (6), (7), and (16) into (9) and (10) we have

$$f_{a_0}(a_0) = \frac{\sqrt{\pi} a_0}{2(\pi/4 + a_0^2)^{3/2}}; \tag{17}$$

$$f_{g_i}(g_i) = \frac{1}{2\sqrt{\pi}(\pi/4 + g_i^2)}. \tag{18}$$

Substituting (16) into (11) and following [13], distribution of $\overline{P}_N$ for the bell shape terminal distribution is:

$$f_{\overline{P}_N}(\overline{P}_N) = \frac{N}{2} \overline{P}_N^{-3/2} \exp\left(-\frac{N^2\pi}{4\overline{P}_N}\right). \tag{19}$$

IV. Probability of Capture

This section provides the exact calculation, an upper bound and a lower bound for the probability of capturing a packet in a collision of $N+1$ packets. We consider BPSK modulation with the probability of bit error P_{be} for the signal-to-noise ratio of γ given by

$$P_{\text{be}}(\gamma) = \frac{1}{2} \operatorname{erfc}(\sqrt{\gamma}).$$

Other modulation techniques and the effects of coding are considered in the next section.

A. The Exact Calculation

For N interfering packets, the probability of bit error for the kth bit in the test packet (the one phase-locked to the receiver) $P_{\text{be}}(N \mid a_0, a_{Nk})$ is found from (5)

$$P_{\text{be}}(N \mid a_{0k}, a_{Nk}) = P(a_{0k} + a_{Nk} + \eta_{0k} < 0 \mid \alpha_{0k} > 0) \tag{20}$$

where $a_0 = a_{0k}$ is the same for all bits in the test packet for the slow-fading channels. Defining the signal to noise ratio as $\gamma = \frac{(a_0 + a_{Nk})^2}{N_0}$ for BPSK modulation, we have

$$P_{\text{be}}(N \mid a_{0k}, a_{Nk}) = \frac{1}{2} \operatorname{erfc}(\sqrt{\gamma}) = \frac{1}{2} \operatorname{erfc}\left(\frac{a_0 + a_{Nk}}{\sqrt{N_0}}\right). \tag{21}$$

For slow fading, the g_{ik} for all bits of a packet are the same, resulting in high correlation between a_{Nk}'s for different bits in the test packet. The change in the interference from one bit to another bit of the test packet is caused by the random pattern of the interfering data bits from other packets. The probability of capture for the test packet of length L with N interfering packets is thus the average over all interfering bit patterns, given by

$$P_C(N \mid a_0, a_{Nk}) = [(1 - E\{P_{\mathrm{be}}(N \mid a_0, a_{Nk})\})]^L = \left(1 - \frac{1}{2}E\left\{\mathrm{erfc}\left(\frac{a_0 + a_{Nk}}{\sqrt{N_0}}\right)\right\}\right)^L \quad (22)$$

where $E\{\}$ represents the averaging over the 2^N possible values of a_{Nk} for different patterns of data α_{ik}. Since $\alpha_{ik} = \pm 1$, this average is given by

$$E\left\{\mathrm{erfc}\left(\frac{a_0 + a_{Nk}}{\sqrt{N_0}}\right)\right\} = \frac{1}{2^N}\sum_{\alpha_{1k}=\pm 1}\cdots\sum_{\alpha_{Nk}=\pm 1} \cdot \frac{1}{2}\mathrm{erfc}\left(\frac{a_0 + \sum_{i=1}^{N}\alpha_{ik}g_i}{\sqrt{N_0}}\right). \quad (23)$$

Given the probability density functions of a_0 and g_i, the average probability of capture for the test packet $P_C(N)$ can be obtained with an $N+1$ dimension integral given by

$$P_C(N) = \int_0^{\infty} da_0 \int_{-\infty}^{\infty} dg_1 \ldots \int_{-\infty}^{\infty} dg_N \cdot f_{a_0}(a_0) f_{g_1}(g_1) \ldots f_{g_N}(g_N) P_C(N a_0, a_N). \quad (24)$$

For the circular and the bell shape terminal distribution used in this paper, the probability density function of a_0 and g_i are given by (13), (14) and (17), (18), respectively.

In this method all possible patterns for the interfering bits are considered, therefore, calculations are exact.

B. The Upper Bound with Correlated Bit Patterns

Assuming $a_{Nk} = a_N$ is constant over all bits in a packet, the results provide an upper bound for the performance. In this case, the interfering bit patterns are assumed to have the same pattern for all bits in a packet, and

$$E\left\{\mathrm{erfc}\left(\frac{a_0 + a_{Nk}}{\sqrt{N_0}}\right)\right\} = \mathrm{erfc}\left(\frac{a_0 + a_{Nk}}{\sqrt{N_0}}\right). \quad (25)$$

Therefore, (22) reduces to

$$P_C(N \mid a_0, a_N) = \left(1 - \mathrm{erfc}\left(\frac{a_0 + a_N}{\sqrt{N_0}}\right)\right)^L. \quad (26)$$

The a_N in this case is a Gaussian random variable with variance $\overline{P}_N/2$. Defining $z = a_0 + a_N$, the distribution of z given $\overline{p}_0$ and $\overline{P}_N$ is the convolution of $f_{a_0}(a_0 \mid \overline{p}_0)$ defined in (6), and $f_{a_N}(a_N \mid \overline{P}_N)$ given by (19)

$$f_z(z \mid \overline{p}_0, \overline{P}_N) = \int_0^{\infty} f_{a_0}(a_0 \mid \overline{p}_0) f_{a_N}((z - a_0) \mid \overline{P}_N) da_0. \quad (27)$$

The probability of capture for a packet with the given $\overline{p}_0$ and $\overline{P}_N$ is then given by

$$P_C(N \mid \overline{p}_0, \overline{P}_N) = \int_{-\infty}^{\infty} P_C(C \mid Z) f_z(z \mid \overline{p}_0, \overline{P}_N) dz. \quad (28)$$

For a known distribution of $\overline{p}$ and $\overline{P}_N$, the average probability of capture for the test packet, given N interfering packets is

$$P_C(N) = \int_0^{\infty}\int_0^{\infty} P_C(N \mid \overline{p}_0, \overline{P}_N) f_{\overline{p}_0}(\overline{p}_0) f_{\overline{P}_N}(\overline{P}_N) d\overline{p}\, d\overline{P}_N = \int_{-\infty}^{\infty} P_C(N \mid z) f_{zN}(z, N)\, dz \quad (29)$$

where

$$f_{zN}(z, N) = \int_0^{\infty}\int_0^{\infty} f_z(z \mid \overline{p}_0, \overline{P}_N) f_{\overline{p}_0}(\overline{p}_0) f_{\overline{P}_N}(\overline{P}_N) d\overline{p}_0\, d\overline{P}_N. \quad (30)$$

C. The Lower Bound with Gaussian Assumption

In a fast-fading channel where the fade rate is faster than the bit rate, the effect of fading on each bit of a packet is independent of the fading effects on the other bits of the same packet; the a_{Nk} for different values of k are then statistically independent. This case provides a lower bound for slotted ALOHA with capture, because the probability of capture of a packet is lower than that of the slow fading.

The calculation of error rate in this case does not include the correlation between the bits of a packet and the statistics of a_{Nk} can be used rather than statistics of all g_i's. The a_{Nk} is a Gaussian random variable with variance $\frac{\overline{P}_N}{2}$, and it affects the detection process the same way as the additive noise. Therefore, the total noise from interference and the channel is Gaussian with variance $(\overline{P}_N + N_0)/2$ and the signal-to-noise and interference ratio is $\gamma = a_0^2/(\overline{P}_N + N_0)$. Since a_0 is Rayleigh, the γ will be exponential of the form

$$f_\gamma(\gamma \mid \overline{\gamma}) = \frac{1}{\overline{\gamma}} \exp\left(-\frac{\gamma}{\overline{\gamma}}\right) \quad (31)$$

where $\overline{\gamma} = \overline{a_0^2}/(\overline{P}_N + N_0) = \overline{p}_0/(\overline{P}_N + N_0)$.

The average probability of error for all bits of the test packet is the same and for a given value of γ, the probability that the test packet will survive is

$$P_C(N \mid \gamma) = (1 - P_{be}(N \mid \gamma))^L. \quad (32)$$

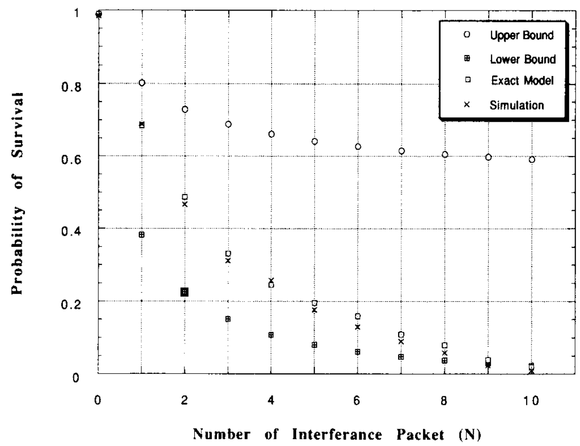

Fig. 1. Probability of capture versus the number of interfering packets for Monte–Carlo computer simulations, exact calculations, upper bound, and the lower bounds. The SNR = 20 dB, the distribution of the terminals is the bell shape distribution, the length of the packets are 16 bits and the modulation is the BPSK.

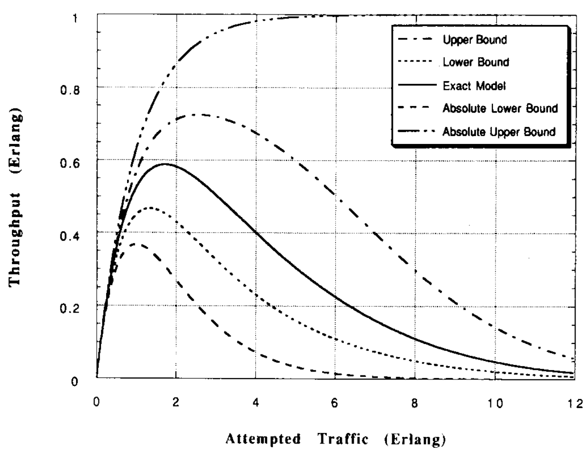

Fig. 2. Throughput versus attempted traffic for the exact calculations, bounds derived from (24) and (34), and the absolute bounds given by (3) and (4). Results of Fig. 1 are used to generate these plots.

The probability of capture for a packet with given $\overline{p}_0$ and $\overline{P}_N$ is then

$$P_C(N\,|\,\overline{\gamma}) = \int_0^{\infty} f_\gamma(\gamma\,|\,\overline{\gamma}) P_C(N\,|\,\gamma)\,d\gamma. \tag{33}$$

Therefore, the probability of capture is

$$P_C(N) = \int_0^{\infty}\int_0^{\infty} P_C\left(N\,|\,\frac{\overline{p}}{\overline{P}_N + N_0}\right) f_{\overline{p}}(\overline{p}) f_{\overline{P}_N}(\overline{P}_N)\,d\overline{p}\,d\overline{P}_N \tag{34}$$

where $f_{\overline{p}}(\overline{p})$ for circular and bell shape distribution are given by (12) and (16), respectively.

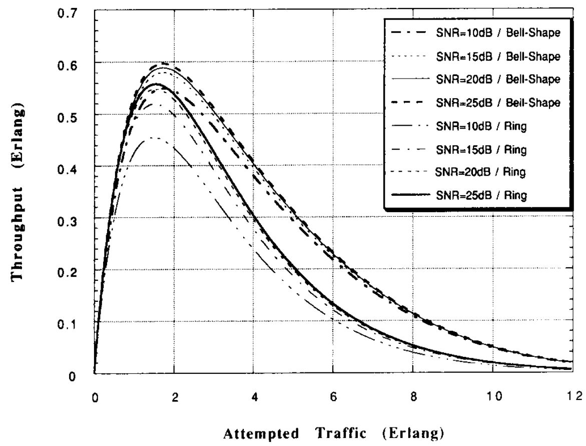

Fig. 3. Exact calculation of the throughput versus the attempted traffic, for different SNR's. Both ring and bell shape distributions are used with BPSK modulation and 16 bits packets.

V. Effects of Modulation and Coding

For other modulations, similar analysis can be applied with appropriate equations for the calculation of the bit error probability. For the coherent frequency shift keying (FSK),

$$P_{\text{be}}(\gamma) = \frac{1}{2}\,\text{erfc}\left(\frac{\gamma}{\sqrt{2}}\right), \tag{35}$$

and for the noncoherent frequency shift keying (NCFSK),

$$P_{\text{be}}(\gamma) = \begin{cases} \frac{1}{2}\exp\left(-\frac{\gamma^2}{2}\right); & \gamma \geq 0 \\ 1 - \frac{1}{2}\exp\left(\frac{\gamma^2}{2}\right); & \gamma < 0. \end{cases} \tag{36}$$

Coding can be used to improve the probability of capture for a packet. We examine the BCH codes to determine the effects of coding in the presence of interfering packets. For a given probability of bit error, the probability of capture of a packet of length L, $P_C(N)$, with block coding is given by [19]

$$P_C(N) = \sum_{i=0}^{t} \binom{L}{i} (1 - P_{\text{be}}(N))^{L-i} (P_{\text{be}}(N))^i \tag{37}$$

where L is the length of the packet and t is the number of bits the error correction code can correct. For the uncoded systems $t = 0$ and the equation reduces to previously used (22).

VI. Results and Analysis

The averaged probability of capture $P_C(N)$ is determined by numerical integration of (24), (29), and (34) for the exact model, the upper bound and the lower bound, respectively. Fig. 1 shows these probabilities of capture and the results of the Monte–Carlo computer simulations for BPSK modulation, SNR = 20 dB, packet size of 16 bits and the bell shape terminal distribution. The horizontal axis is the number of interfering packets and the SNR is defined for a terminal located at the median of the distances of the terminals from the central station. The results of the exact calculation and computer simulation show a close agreement.

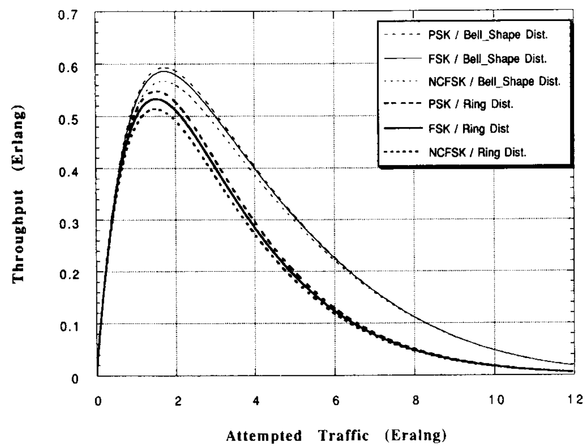

Fig. 4. Exact calculation of throughput versus the attempted traffic, for different lengths of the packet. Both ring and bell shape distributions are shown with BPSK modulation and SNR = 20 dB.

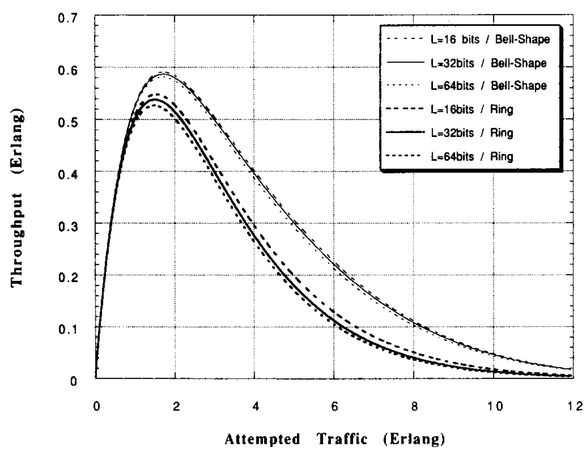

Fig. 5. Exact calculation of throughput versus the attempted traffic, for different modulation techniques. Both ring and bell shape distributions are shown with 16 bits packets and SNR = 20 dB.

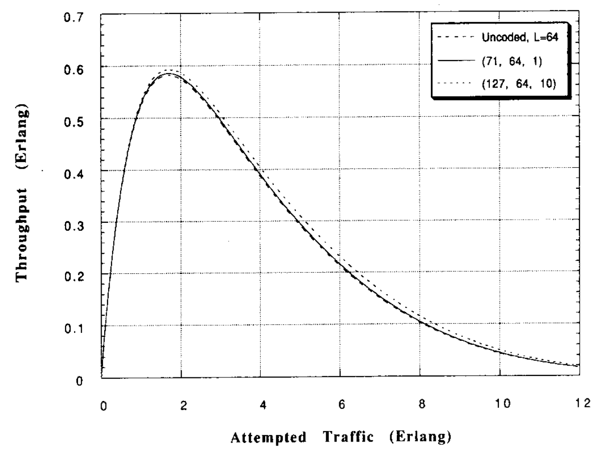

Fig. 6. Exact calculation of the throughput versus attempted traffic, for uncoded BPSK, $(71,64,1)$ and $(127,64,10)$ BCH codes with SNR = 20 dB and the bell shape distribution of the terminals.

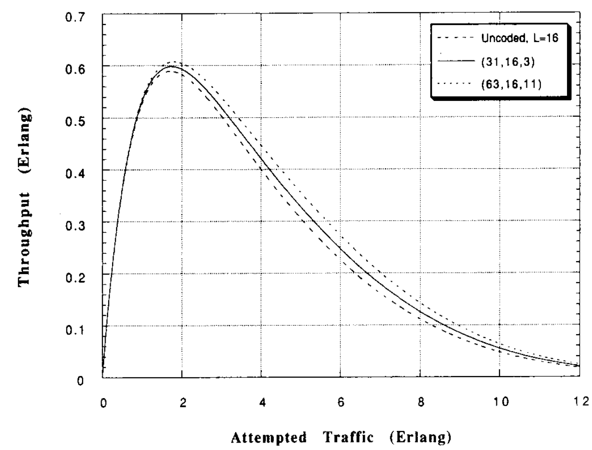

Fig. 7. Exact calculation of the throughput versus attempted traffic, for uncoded BPSK, $(31,16,1)$ and $(63,16,11)$ BCH codes with SNR = 20 dB and the bell shape distribution of the terminals.

Fig. 2 shows the relation between average throughput S and the attempted traffic G for different bounds and the exact calculations. The absolute bounds are determined from (3) and (4), and the exact calculations and the other two bounds are found by substituting the results of Fig. 1 in (2). The upper bound using (24) and the lower bound using (34) are significantly tighter than the absolute upper and lower bounds given by (3) and (4), respectively.

Fig. 3 shows the relation between average throughput S, and the attempted traffic G for packet length of 16 bits and BPSK modulation, with different SNR. The maximum effect of the SNR is observed for the ring distribution, which accounts for the approximate 18% drop in the peak throughput when the SNR is reduced from 25 to 10 dB. Due to the wide variations of the average received power for the bell shape terminal distribution, the bell shape distribution is less sensitive to the SNR, when compared to the ring distribution.

Fig. 4 shows the relation between the average throughput S, and the attempted traffic G for different packet lengths. The maximum effect is observed for the ring distribution where the throughput of the 64 bit packet system is about 8% less than a 16 bit packet system. Fig. 5 shows the effects of modulation technique on the throughput of the slow fading channels with packet size of 16 bits, SNR = 20 dB and the PSK, FSK, and NCFSK modulations. The maximum effect on the throughput is observed for the ring distribution where the maximum throughput with the PSK modulation is 5% higher than the maximum throughput with the NCFSK modulation. Fig. 6 shows the effects of coding on the throughput in the slow fading channels, for a 64 bit packet coded into 71 bit and 127 bit packets, respectively. BCH coding, PSK modulation, and the SNR of 20 dB are considered in this figure. Fig. 7 is similar to Fig. 6, with 16 bit packets coded into 31 bits and 63 bits, respectively. The coding shows a minimal effect on the throughput because the channel is slow fading and if it is in a deep fade the whole packet is destroyed and if it is out of fade the packet survives anyhow; with or without coding.

VII. Conclusion

An exact analysis, a lower bound and an upper bound for the throughput of the local packet radio networks using slotted ALOHA was presented. It was shown that, for a system with the ring and bell shape terminal distribution in the area, a throughput as high as 60% is expected for slow fading channels. Comparing the bell shape terminal distribution with the ring distribution, the bell shape is less sensitive to all parameters considered, and it provides a higher throughput for the parameters analyzed. The size of the packets, the SNR, and the modulation and coding techniques showed a maximum of 18% change in the throughput of the system, observed for the ring distribution.

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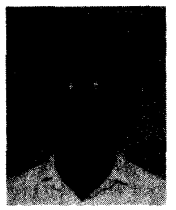

Ker Zhang was born in Canton, China, on October 1962. He received the B.S. degree in physics from Jinan University, China, in 1982, the M.S. degree in applied physics from the University of Massachusetts in 1985, and the Ph.D. degree in electrical engineering from Worcester Polytechnic Institute in 1990.

Following graduation from WPI, he joined the cellular division of Motorola Inc., Arlington, Heights, IL. Since October 1991, he has been with the digital communication division of Rockwell International, Newport Beach, CA, where he is engaged in research and development for personal communication systems. His research interests include digital cellular system, wireless indoor network and integrated voice and data communication over fading channels.

Dr. Zhang is a member of Eta Kappa Nu and American physics society.

Kaveh Pahlavan (M'79–SM'88) was born in Teheran, Iran, on March 16, 1951. He received the M.S. degree from the University of Teheran, Teheran, Iran, in 1975 and the Ph.D. degree from the Worcester Polytechnic Institute, Worcester, MA, in 1979.

From 1979 to 1983, he was an Assistant Professor in the Electrical and Computer Enginering Department, Northeastern University, Boston, MA. He was consulting with CNR, Inc., Needham, MA, and GTE Laboratories, Waltham, MA, during the same period. He joined Infinet, Inc., in 1983 and served as the senior staff engineer, and later on, as the director of advanced developments until 1985. During this period, he was teaching DSP courses at the graduate scool of Northeastern University. In September 1985, he joined the faculty of Worcester Polytechnic Institute, where he is currently a Professor of Electrical Engineering and the Director of the Center for Wireless Information Network Studies (CWINS). During services at WPI, he has been a consultant to GTE Laboratories, Waltham, MA, Steinbrecher Associates, Woban, MA, Simplex, Inc., Gardner, MA, and WINDATA, Inc., Northborough, MA, in various aspects of wireless indoor networks. His present areas of interest are all related to wireless information networks. His background research is on voice-band data communications and fading channel communications. He has contributed in numerous papers, two patents, a chapter in a book, and has given tutorial talks in various countries.

Dr. Pahlavan was the Founder and Chairman of the IEEE Workshop on Wireless Local Area Network held at WPI on May 9–10, 1991. He is the organizer and technical program chairman of the IEEE Third Symposium on Portable, Indoor and Mobile Radio Communications, to be held in Boston, MA, October 19–21, 1992. He has also served in the program committee of several related conferences. He is a member of Eta Kappa Nu.